A Microwave Ferrite Frequency Separator*

HAROLD RAPAPORT†

Summary—**When multiple filter groups are interconnected for operation out of a single source, interaction effects between filters can occur. Frequently, unless special precautions are taken, the filters may interact to such an extent that severe deterioration in performance may result. Introduction of the gyrator by Tellegen and the subsequent microwave realization of the circulator by Hogan, Rowen, and others provide new possibilities for design of channel-branching circuits and frequency-spectrum partition arrays.**

The nature of the frequency separation problem is reviewed, and the application of the ferrite circulator to effect channel branching is considered in detail. Several specific multichannel systems comprising various circulator filter and one-way line filter arrays are presented and their relative merits examined.

A 4-port (3-channel) experimental prototype separator system consisting of a Faraday rotation type of circulator and maximally flat band-pass waveguide filters is described. A quantitative theory of operation of the prototype is developed. Experimental data and performance curves are given. These data show close agreement with results predicted by the theory.

General Considerations in Applying Frequency Partition Filters

Nature of The Frequency Separation Problem

It is well known that when multiple filter groups are interconnected for operation out of a single source, interaction effects will occur between the individual filters.[1] In general, unless special precautions are taken, the filters may interfere with each other to such an extent that severe deterioration in performance throughout the entire frequency region of interest may result.

In the past, various techniques have been evolved in efforts to eliminate or at least minimize filter interaction. The general approach to the solution of the interaction problem lies in the design and synthesis of complementary pairs of filters, by techniques based on the methods developed by Zobel,[2] Norton,[3] and Bode.[4] In the low-frequency region, where line lengths are nonexistent and lumped elements are available, these techniques offer a reasonable solution to most "channel-branching" problems, provided the required number of channels is not too large. However, as the frequency is increased and the transmission-line nature of the problem must be considered, the utility of these techniques is substantially diminished.

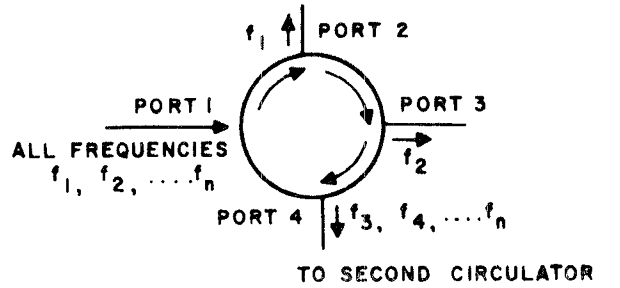


Fig. 1—Diagrammatic representation of frequency separation by means of a circulator.

An investigation into the general properties of gyrator networks at the Microwave Research Institute by Rudner[5] and Carlin[6] indicated the potential application of the circulator principle to frequency separation problems. Consider the idealized 4-port circulator configuration as represented in Fig. 1.

Recalling the basic defining characteristic of the circulator; all energy incident on port 1 will be coupled to port 2, all energy incident on port 2 will couple to 3, energy into 3 will couple to 4, and energy into 4 will couple to 1—for the port designations used in the figure. In the ideal case, no reverse coupling or cross-coupling will exist. If, for the present, ideal behavior is assumed, the application of the circulator to the frequency separation problem is immediately apparent. As indicated in the figure, if a number of frequencies $f_1, f_2, \cdots, f_n$ are considered incident on port 1, they will all be transmitted to port 2 by virtue of the circulator action described above. Consider a filter located at port 2, with a frequency characteristic such that only f_1 (or a small band about f_1) will be passed.

It is clear that f_1 will be removed at port 2, but all other frequencies will be reflected and will proceed to port 3. If a second filter permitting passage only of f_2 is located at port 3, then only f_2 will be removed while $f_3, f_4, \cdots, f_n$ will be reflected to port 4. If provision is made at port 4 for the transmission of all of the remaining frequencies, they may be passed on to a second circulator in which further separation can be accomplished. Other arrangements of circulators and filters may be used, and several special cases will be considered later.

Quantitative Aspect

Some of the fundamental quantitative relationships concerning the operation of a 4-port circulator filter fre-

* Manuscript received by the PGMTT, June 17, 1957. The work reported here was supported by the Electronic Warfare Laboratory, Rome Air Dev. Ctr. Contract No. AF-30(602)-981, and was carried out at the Microwave Research Institute, Polytechnic Institute of Brooklyn, when the author was a member of the staff.

† Defense Electronics Products, RCA, New York, N. Y.

[1] E. A. Guillemin, "Communication Networks," John Wiley & Sons, Inc., New York, N. Y., vol. 2, pp. 356–375; 1935.

[2] O. J. Zobel, "Distortion correction in electrical circuits with constant resistance recurrent networks," *Bell Sys. Tech. J*, vol. 7, pp. 438–534; July, 1928.

[3] E. L. Norton, "Constant Resistance Networks with Applications to Filter Groups, Bell Sys. Monograph B-991; 1937.

[4] H. W. Bode, "A Method of Impedance Correction," *Bell Sys. Tech. J.*, vol. 9, pp. 794–835; October, 1930.

[5] B. Steinman, "Analysis of Some Microwave Ferrite Devices," Microwave Res. Inst., Polytech. Inst. of Brooklyn, Rep. R-367-53, PIB-301; February, 1954.

[6] H. J. Carlin, "Theory and Applications of Gyrator Networks," Microwave Res. Inst., Polytech. Inst. of Brooklyn, Final Rep. R-355-53, PIB-289; March, 1954.

quency separator will be developed. The scattering formulation of the network problem[7,8] will be used. Port 1 of the circulator is taken as input.

In terms of the voltage scattering parameters, the network equations for a 4-port are

$$\left.\begin{aligned} v_{r1} &= S_{11}v_{i1} + S_{12}v_{i2} + S_{13}v_{i3} + S_{14}v_{i4} \\ v_{r2} &= S_{21}v_{i1} + S_{22}v_{i2} + S_{23}v_{i3} + S_{24}v_{i4} \\ v_{r3} &= S_{31}v_{i1} + S_{32}v_{i2} + S_{33}v_{i3} + S_{34}v_{i4} \\ v_{r4} &= S_{41}v_{i1} + S_{42}v_{i2} + S_{43}v_{i3} + S_{44}v_{i4} \end{aligned}\right\} \qquad (1)$$

in which the v_{rm} represent the reflected voltages (energy flow polarized *out* of the network), the v_{in} represent the incident voltages (energy flow polarized *into* the network), and the S_{mn} are the complex voltage transfer coefficients of the scattering matrix (reflection coefficients if $m=n$). The S_{mn} have the form, $S_{mn}=|S_{mn}|e^{j\theta_{mn}}$.

If structures defined by input reflection coefficients K_2, K_3, and K_4 terminate circulator ports 2, 3, and 4, respectively, then

$$\left.\begin{aligned} v_{i2} &= K_2 v_{r2} \\ v_{i3} &= K_3 v_{r3} \\ v_{i4} &= K_4 v_{r4} \end{aligned}\right\}. \qquad (2)$$

Substituting the set of (2) into (1) and solving for the voltage ratios v_{rm}/v_{i1}

$$\frac{v_{r1}}{v_{i1}} = \frac{\begin{vmatrix} -S_{11} & S_{12}K_2 & S_{13}K_3 & S_{14}K_4 \\ -S_{21} & (S_{22}K_2-1) & S_{23}K_3 & S_{24}K_4 \\ -S_{31} & S_{32}K_2 & (S_{33}K_3-1) & S_{34}K_4 \\ -S_{41} & S_{42}K_2 & S_{43}K_3 & (S_{44}K_4-1) \end{vmatrix}}{|D|} \qquad (3)$$

$$\frac{v_{r2}}{v_{i1}} = \frac{\begin{vmatrix} -1 & -S_{11} & S_{13}K_3 & S_{14}K_4 \\ 0 & -S_{21} & S_{23}K_3 & S_{24}K_4 \\ 0 & -S_{31} & (S_{33}K_3-1) & S_{34}K_4 \\ 0 & -S_{41} & S_{43}K_3 & (S_{44}K_4-1) \end{vmatrix}}{|D|}, \qquad (4)$$

$$\frac{v_{r3}}{v_{i1}} = \frac{\begin{vmatrix} -1 & S_{12}K_2 & -S_{11} & S_{14}K_4 \\ 0 & (S_{22}K_2-1) & -S_{21} & S_{24}K_4 \\ 0 & S_{32}K_2 & -S_{31} & S_{34}K_4 \\ 0 & S_{42}K_2 & -S_{41} & (S_{44}K_4-1) \end{vmatrix}}{|D|}, \qquad (5)$$

$$\frac{v_{r4}}{v_{i1}} = \frac{\begin{vmatrix} -1 & S_{12}K_2 & S_{13}K_3 & -S_{11} \\ 0 & (S_{22}K_2-1) & S_{23}K_3 & -S_{21} \\ 0 & S_{32}K_2 & (S_{33}K_3-1) & -S_{31} \\ 0 & S_{42}K_2 & S_{43}K_3 & -S_{41} \end{vmatrix}}{|D|}, \qquad (6)$$

[7] H. J. Carlin, "An Introduction to the Use of the Scattering Matrix in Network Theory," Microwave Res. Inst., Polytech. Inst. of Brooklyn, Rep. R-366-54, PIB-300; June, 1954.

[8] G. L. Ragan, "Microwave Transmission Circuit," Mass. Inst. Tech. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 9, pp. 551–554; 1948.

where

$$|D| = \begin{vmatrix} -1 & S_{12}K_2 & S_{13}K_3 & S_{14}K_4 \\ 0 & (S_{22}K_2-1) & S_{23}K_3 & S_{24}K_4 \\ 0 & S_{32}K_2 & (S_{33}K_3-1) & S_{34}K_4 \\ 0 & S_{42}K_2 & S_{43}K_3 & (S_{44}K_4-1) \end{vmatrix}. \qquad (7)$$

The voltage ratios of (4)–(6) may be defined as circulator coefficients $C_{m1}=V_{rm}/v_{i1}$, associated with the loaded circulator and referred to the voltage incident on the input terminals of the terminating structures.

In the case under discussion, since terminations associated with the various ports are filters (2-port structures), the quantities of primary interest are the insertion loss from port 1 to the *output terminals* of the mth filter and, in certain instances, the input vswr measured at port 1. Since each filter is a 2-port structure, the scattering equations for the mth filter will have the form

$$\left.\begin{aligned} v_{r1}' &= [T_{11}]_m v_{i1}' + [T_{12}]_m v_{12}' \\ v_{r2}' &= [T_{21}]_m v_{i1}' + [T_{22}]_m v_{12}' \end{aligned}\right\} \qquad (8)$$

in which the numerical subscripts 1 and 2 define the input and output terminals respectively of the *terminating structure*, and $[T]_m$*'s* are the scattering coefficients of the filter.

Filter performance is generally defined in terms of the insertion loss (or gain) of the filter when operating between matched terminations. Under these conditions, the value of v_{12}' is zero, so that

$$v_{r1}' = [T_{11}]_m v_{i1}' \qquad (9a)$$

$$v_{r2}' = [T_{21}]_m v_{i1}' \qquad (9b)$$

Now, from (9a) the coefficient T_{11} is seen to be the input reflection coefficient of the terminating structure associated with the mth circulator port. Further, the voltage *incident on* the input terminals of the mth filter is that *reflected out of* the circulator. Thus,

$$K_m = [T_{11}]_m \qquad (10a)$$

$$v_{rm} = v_{i1}' \qquad (10b)$$

where the v_{i1}' is understood to refer to the mth port terminating structure as implied by K and v_{rm}.

The voltage measured at the filter output terminals may now be expressed in terms of the input voltage at port 1 and the circulator filter coefficients. Using the C_{m1} of (4), (5), and (6), and combining (9b) and (10b)

$$v_{r2}' = [T_{21}]_m v_{rm} = [T_{21}]_m C_{m1} v_{i1}. \qquad (11)$$

Assuming the same normalizing impedance for both port 1 and the termination of the mth filter, the insertion loss from port 1 to the mth output terminals may be written as

$$L_{m1} = 10 \log \left| \frac{v_{i1}}{v_{r2}'} \right|^2 \text{db}. \qquad (12)$$

Using (11) in (12)

$$L_{m1} = 10 \log \frac{1}{|T_{21}|_m^2 |C_{m1}|^2} \text{ db} \qquad (13)$$

where the C_{m1} are obtained from (4)–(6) using the K_m as defined by (10a). Since the insertion loss of the *m*th filter alone, L_{Fm}, is by definition

$$L_{Fm} = 10 \log \frac{1}{|T_{21}|_m^2} \text{ db.} \qquad (14)$$

Eq. (13) may be written as

$$L_{m1} = 10 \log \frac{1}{|T_{21}|_m^2} + 10 \log \frac{1}{|C_{m1}|^2} \text{ db.} \qquad (15)$$
$$= L_{Fm} + L_{cm} \text{ db,}$$

where the L_{cm} is understood to refer to port 1 as input.

Expressing the total loss in the form of (15) shows that the insertion loss for the interconnected circulator filter combination is a simple sum, and that the entire problem of interaction may be discussed in terms of the *variation* of the function L_{cm}.

The input vswr as measured at port 1 is quickly determined since by definition

$$\text{vswr} = \frac{1 + |K|}{1 - |K|}, \qquad K = \frac{v_{r1}}{v_{i1}}. \qquad (16)$$

To illustrate the use of the equations developed above, the case of a 4-port circulator-filter combination will be discussed. The circulator is assumed to be matched but lossy, and to couple in a 1→2→3→4→1 pattern.

As may be shown,[5,6] the voltage scattering matrix for a perfect circulator of the Faraday rotation type with a 1→2→3→4→1 coupling pattern is

$$S^v = e^{j\theta} \begin{vmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}, \qquad \theta = \alpha + j\beta. \qquad (17)$$

Using the S values as defined by (17) in (4)–(6) and taking the absolute values of the resulting voltage ratios

$$\left|\frac{v_{r1}}{v_{i1}}\right| = |K_2||K_3||K_4| e^{4\alpha}, \qquad (18)$$

$$C_{21} = \left|\frac{v_{r2}}{v_{i1}}\right| = e^{\alpha} \qquad (19)$$

$$C_{31} = \left|\frac{v_{r3}}{v_{i1}}\right| = |K_2| e^{2\alpha}, \qquad (20)$$

$$C_{41} = \left|\frac{v_{r4}}{v_{i1}}\right| = |K_2||K_3| e^{3\alpha}. \qquad (21)$$

These results show that the energy input into port 1 is not equally available to all terminations. Clearly, from (19) all energy except that dissipated in the ferrite is available to the structure terminating port 2. But (20) shows that energy at port 3 will depend on the input reflection factor of the port 2 structure. In the same way, (21) indicates energy available at port 4 will be a function of the reflections of both the port 2 and the port 3 structures. However, it should be stressed that the energy availability *does not* depend on the *relative phases* of the reflections. Only the *magnitudes* of the filter reflection factors enter into the interaction.

Using (19)–(21) in (15), the loss quantities, L_{cm}, are seen to be

$$L_{c2} = 10 \log e^{2\alpha}, \qquad (22)$$

$$L_{c3} = 10 \log \frac{1}{|K_2|^2} = 10 \log \frac{1}{[T_{11}]_2^2} + \log e^{4\alpha}, \qquad (23)$$

$$L_{c4} = 10 \log \frac{1}{|K_2|^2 |K_3|^2}$$
$$= 10 \log \frac{1}{[T_{11}]_2^2} + 10 \log \frac{1}{[T_{11}]_3^2} 10 \log e^{6\alpha}. \qquad (24)$$

These results demonstrate that in the case of the perfect circulator, the *variation* in insertion loss as measured at the output terminals of a given filter will depend *only* on the magnitudes of the reflection coefficients of the filters *preceding* it in the circulator coupling pattern. For example, (22) indicates that *regardless* of the terminations on all *succeeding* ports, the *variation* in insertion loss will be zero. Thus, the total insertion loss, L_{21}, will equal the original filter loss characteristic, L_{F2}, plus the circulator dissipative loss.

Eqs. (23) and (24) indicate that if the interaction effects are to be further evaluated, magnitudes of the input reflection factors of the various filters must be specified. For any lossless 2-port structure it may be shown[7,8] that

$$|T_{11}|^2 = 1 - |T_{21}|^2 \qquad (25)$$

so that if the filters are lossless, (23) and (24) become

$$L_{c3} = 10 \log \frac{1}{1 - [T_{21}]_2^2} + 10 \log e^{4\alpha}, \qquad (26)$$

$$L_{c4} = 10 \log \frac{1}{1 - [T_{21}]_2^2} + 10 \log \frac{1}{1 - [T_{21}]_3^2}$$
$$+ 10 \log e^{6\alpha}. \qquad (27)$$

These equations allow calculation of the interaction effects from knowledge of the insertion loss characteristics of the filters (if lossless), rather than knowledge of the input vswr as required in (23) and (24).

At this point it is necessary to define explicitly either the $|K|$'s of (23) and (24) or, if lossless, the $|T_{21}|$'s of (26) and (27). For purposes of further discussion, the $|K|$'s will be assumed to be related in the following sense. When the frequency is such that the filter located at port 2 is in the pass band ($|K_2| = 0$), then the filters at ports 3 and 4 will be assumed to be completely rejecting ($|K_3| = |K_4| = 1$). Similarly when $|K_3| = 0$, then $|K_2| = |K_4| = 1$. Finally when $|K_4| = 0$, then $|K_2| = |K_3| = 1$. This situation is characteristic of the design of most channel branching systems in that a given branching filter will present a high rejection to frequencies which lie in the pass bands of neighboring filters. Thus, for the 4-port 3-filter system being considered, under these restrictions (23) and (24) show that

$$L_{c3} = 10 \log \frac{1}{|K_2|^2} = 10 \log e^{4\alpha},$$

$$L_{c4} = 10 \log \frac{1}{|K_2|^2 |K_3|^2} = 10 \log e^{6\alpha}. \qquad (28)$$

Thus, in the *pass bands* of the various filters, no interaction effects will occur, and the total insertion loss to the filter output terminals will be that produced by the filter alone plus circulator dissipation.

The reflections at port 1 will be zero since some $|K_k| = 0$. Thus, the vswr measured at port 1 will be unity and the system will appear matched. It should also be observed that if a matched load is placed at one of the ports, say port 4, rather than a filter, the system will appear matched at *all frequencies*—not only in the pass bands. This property may be quite desirable in certain systems applications in which the mismatch seen by the generator is of concern.

The interaction effects in those frequency regions in which the $|K|$ restrictions given above no longer apply cannot be summarized so simply. As (26) and (27) indicate, the extent of the interaction will depend on the explicit form and relationship of the filter functions considered. The interaction effects in these "crossover frequency" regions can be analytically treated but space does not permit further consideration of these effects here.

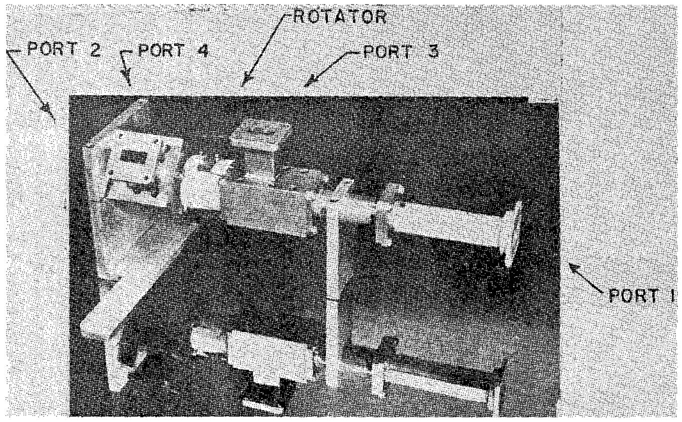
Experimental Prototype Frequency Separator

In order to experimentally verify the preceding analysis, an experimental prototype separator was designed and constructed in the 9.0 kmc frequency region. The separator was of the Faraday rotation type described by Hogan[9] and others.[10] The filters were band-pass with a maximally-flat characteristic and realized in rectangular waveguide. The filters were 100 mc wide at the 3 db points and had center frequencies of 8800, 8900, and 9000 mc. These elements of the prototype separator are shown in Fig. 2.

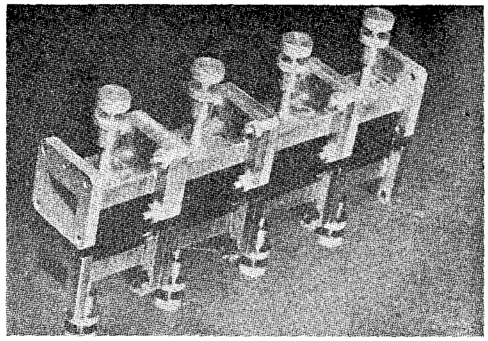
Briefly, the experimental characteristics pertinent to the present discussion are that the circulator exhibited a loss of 0.6 db and the filters a loss of 0.8 db in their pass bands.

[9] C. L. Hogan, "The ferromagnetic effect at microwave frequencies and its applications-the microwave gyrator," *Bell Sys. Tech. J.*, vol. 31; January, 1952.

[10] J. H. Rowen," "Ferrites in microwave applications," *Bell Sys. Tech. J.*, vol. 32; November, 1953.



(a)



(b)

Fig. 2—Frequency separator components. (a) Circulator, (b) filter.

The characteristics of the associated filters alone are shown in Fig. 3. Here the experimentally measured insertion loss values have been superimposed on a frequency scale to show the crossover points. The midband losses are 0.8 db. It is seen that the filters are essentially identical.

In Fig. 4 the insertion losses to the outputs of these same filters, now located at circulator ports 2, 3, and 4 respectively, are shown again superimposed. This port-filter arrangement was chosen so that the increase in insertion-loss level produced by multiple transits of the ferrite would appear as "step-wise increasing" with increasing frequency in the data display. This increase in level was exhibited by the $e^{m\alpha}$ of the preceding analysis. Note that the leading edge of the insertion loss curves of the filters located at ports 3 and 4 are distorted.

This is an example of the interaction effect previously mentioned. The experimental results and analytical predictions agree fairly well for these distortions.

Shown in Table I is a tabulation of the pass band losses predicted by (15) and those measured experimentally. As the table shows, the agreement here is quite good.

Some General Frequency Partition Systems

The present model of the prototype frequency separator has been shown to function adequately in a 3-channel branching application, and the experimental results in-

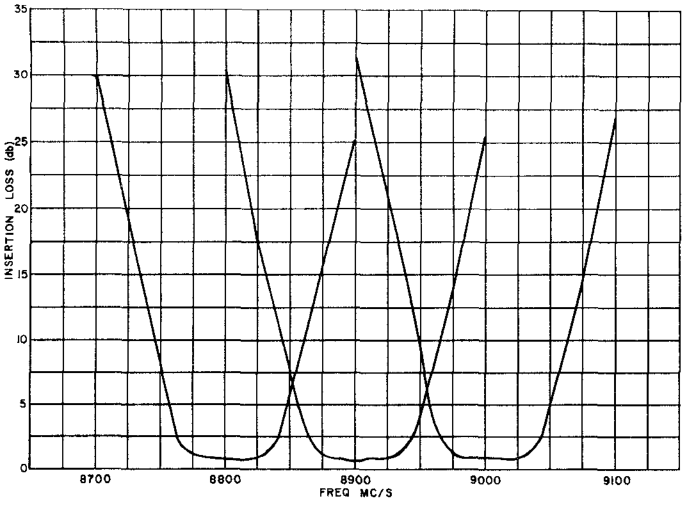


Fig. 3—Insertion loss of associated filters.

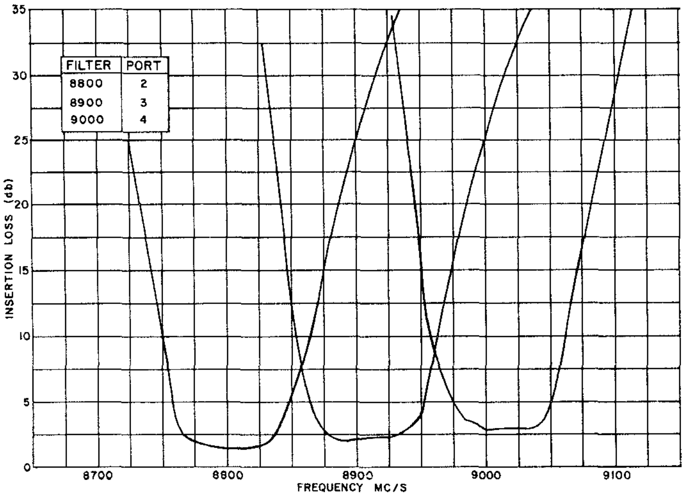


Fig. 4—Prototype frequency separator experimental results.

TABLE I

FREQUENCY SEPARATOR PASS BAND LOSSES

Port	L_{Fm}db	L_{cm}db	L_{m1}(pred.) db	L_{m1}(meas.) db
2	0.8	0.6	1.4	1.5–1.6
3	0.8	1.2	2.0	1.9–2.0
4	0.8	1.8	2.6	2.7–2.8
		$L_{m1}=L_{Fm}+L_{cm}$		

dicate the validity of the approximate theory developed However, it should not be construed that this *type* of circulator-filter arrangement is unique and must necessarily be used for channel-branching. The predictive aspects of the theory developed above should be capable of extension to other types of possible circulator-filter channel-branching arrays. As an illustration, the pass-band behavior of three possible arrays for a p-channel system will be considered. For simplicity any interconnecting lines or filters will be considered perfect, *i.e.* dissipationless and reflectionless in the pass band regions. The three systems are shown schematically in Fig. 5.

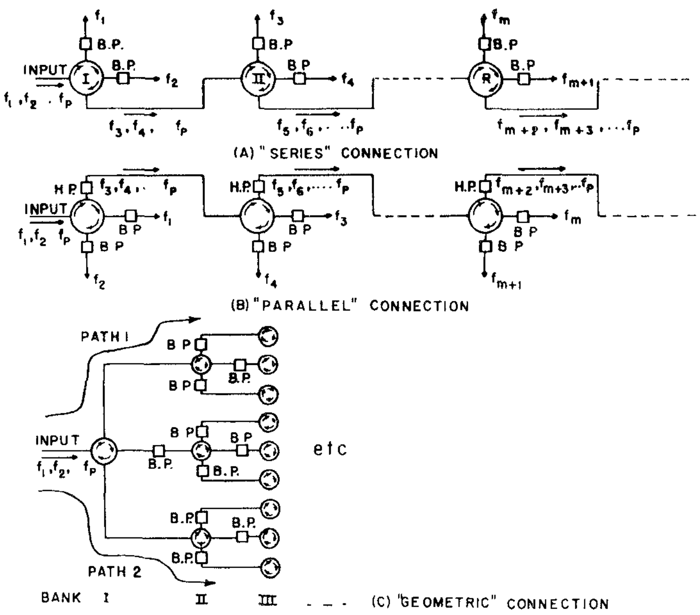


Fig. 5—Three possible branching arrays for frequency separation.

"Series" Connection

The first branching array, shown in Fig. 5(a), is similar to the system used in the prototype separator in that separation is realized by a set of sequentially arranged band-pass filters. Port 1 of circulator I is taken as input, and the arrows indicate the path of energy transmission between preferred ports. This array may be considered to be a "series" connection in the sense that prior to arrival at the proper filter, any energy input into the system must traverse *all* preceding circulators and be available to *all* preceding filters.

"Parallel" Connection

The branching array shown in Fig. 5(b) may be viewed as a "parallel" connection in that it is not necessary for input energy to traverse each circulator in its entirety before arrival at the proper filter.

It should be clear that the particular filter configuration used, *i.e.* high-pass and band-pass, is not the only possible arrangement for this type of service.

"Geometric" Connection

From its similarity to a geometric progression, the third branching array shown in Fig. 5(c) may be viewed as a "geometric" interconnection. The circulators may be considered to be grouped into a series of "banks".

Space does not permit a complete exposition of the performance of each of these systems. The pertinent characteristics are summarized in Table II. The performance can be compared in terms of the circulator loss function $U_{m1}A$; where A is the dissipative loss in the ferrite and U_{m1} is as shown in the table.

It is to be noted that while the required number of circulators is independent of the type of interconnection, the table shows that the ferrite loss function, U_{m1}, is *extremely sensitive* to the form of the array. From the

TABLE II
PASS BAND LOSSES OF THREE FREQUENCY SEPARATOR SYSTEMS

	Series	Parallel	Geometric
U_{m1}	$\frac{3m-2}{2}$ m even; $\frac{3m-1}{2}$ m odd	$\frac{m+4}{2}$ m even; $\frac{m+3}{2}$ m odd	b to $3b$; $b+1$ to $3(b+1)$
C_p	$\frac{p}{2}$ p even; $\frac{p-1}{2}$ p odd	$\frac{p}{2}$ p even; $\frac{p-1}{2}$ p odd	$\frac{p}{2}$ p even; $\frac{p-1}{2}$ p odd
		$L_{m1}=L_{Fm}+L_{cm}$ where $L_{cm}=U_{m1}A$	

point of view of minimum ferrite loss (especially for large p), the geometric type of branching array is superior.

As may be seen from the table, for any given p the required number of circulators, C_p, is always the same, *i.e. independent* of the type of connection. This result may be viewed as a practical expression of the following gyrator network theorem recently established by H. J. Carlin of the Microwave Research Institute.

Theorem: The minimum number of gyrators necessary for the realization of an n-port (n-terminal-pair) circulator is given by

$$G_m = \begin{cases} \frac{1}{2}(n-1) & n \text{ odd,} \\ \frac{1}{2}(n-2) & n \text{ even,} \end{cases}$$

where G_m is the minimum number of necessary gyrators and n is the number of terminal pairs. It may be further stated that the minimum number realization can always be effected for any n by the interconnection of only 3-port and 4-port circulators, each type of which requires only one gyrator.

From the point of view of the theorem, each of the three systems previously described is equivalent to an n-port "minimum-number" type of interconnection in which $n=p+1$. Thus, since each circulator provides one gyrator, it immediately follows from the theorem that the minimum number of necessary circulators will be as shown below.

$$C_p = \begin{cases} \left(\frac{n-1}{2}\right) = \frac{p}{2} & n \text{ odd, } p \text{ even,} \\ \left(\frac{n-2}{2}\right) = \frac{p-1}{2} & n \text{ even, } p \text{ odd.} \end{cases}$$

The results for C_p tabulated in Table II represent special cases of this general result and there is no other possible connection which can use a smaller number of components.

ACKNOWLEDGMENT

The author wishes to acknowledge the helpful advice of Dr. H. J. Carlin of the Microwave Research Institute, and also the efforts of Carl Bollinger in making many of the measurements.